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It is believed that the average probable error of the paralaxes of these nine clusters is much less than 15%. Suppose, however, that for one of the clusters the true distance differs by 30% from the adopted value. The adopted absolute magnitudes for that cluster would be systematically in error by about 0.6 mag. The position of the maxima of the luminosity curves for the nine clusters together would be slightly altered, therefore, if the supposed error were uncompensated; but the general form of none of the curves would be materially changed.

Figure 5 represents the distribution in color and luminosity of the individual stars, the circles representing data from Messier 11, the dots giving results for the most accurately measured clusters (Messier 3, 5, 13, 15, and 68), and the crosses representing the stars for the clusters for which the magnitudes depend upon less extensive investigations. Together with the luminosity and color curves, this diagram illustrates the present state of our information concerning the giant stars in clusters.

ON THE DISTORTION IN CONFORMAL MAPPING WHEN THE SECOND COEFFICIENT IN THE MAPPING FUNCTION HAS AN ASSIGNED VALUE

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Communicated by E. H. Moore, April 27, 1920

Note III On Conformal Mapping Under Aid of Grant No. 207 From the Bache Fund

Let $w = z + a_2z^2 + \ldots + a_nz^n + \ldots$ be a power series in z convergent for $|z| < 1$ and such that the circle $|z| < 1$ is mapped conformally on a simple (that is, simply connected and nowhere overlapping) region in the w-plane. Koebe[1] has shown that on the circumference $|z| = r$, where $0 < r < 1$, the distortion $|dw/dz|$ and also $|z|$ lie between positive bounds depending on r alone, and the writer[2] has determined the exact values of these bounds. A far more difficult problem arises when some of the coefficients of the power series are given *a priori*. The simplest case where $a_2 = ae^{\gamma i}$ $(a \geq 0)$ is given was investigated by the writer,[3] the method employed failing, however, to furnish the upper bound of $|z|$ in the case $0 \leq a < 1$. This defect has now been remedied, and denoting by $r(a)$, for $0 \leq a < 1$, the root between zero and unity of the equation

$$\frac{2r}{1 + 2(a-1)r + r^2} - \log \frac{1 + r}{1 - r} = 0,$$

and by $\cos \beta$, for $0 < r \leq r(a)$, the positive root of the equation

$$\frac{2r}{1 - 2r\cos \beta + r^2} - \log \frac{1 + r}{1 - r} = 0,$$

we have the following:

THEOREM: *When the analytic function*

$$w = z + ae^{\gamma i}z^2 + a_3 z^3 + \ldots + a_n z^n + \ldots$$

(where $a \geqq 0$ and γ real) maps the circle $|z| < 1$ on the interior of a simple region D in the w-plane, then $a \leqq 2$, and we have, for $|z| = r$ and $0 < r < 1$,

$$\frac{1 - r^2}{(1 + ar + r^2)^2} < \left|\frac{dw}{dz}\right| < \begin{cases} \dfrac{1 - 2r\cos\beta + r^2}{(1 - r)^{2+\cos\beta+a}(1 + r)^{2-\cos\beta-a}} \\ \text{[for } a < 1 \text{ and } r \leqq r(a)\text{]}, \\ \dfrac{1 + 2(a - 1)r + r^2}{(1 - r)^3(1 + r)} \\ \text{[for } a < 1 \text{ and } r \geqq r(a)\text{, and for } 1 \leqq a \leqq 2\text{]}, \end{cases}$$

$$\frac{r}{1 + ar + r^2} < |w| < \begin{cases} \dfrac{1}{4(1+a)}\left[\left(\dfrac{1+r}{1-r}\right)^{1+a} - 1\right] + \dfrac{1}{4(1-a)}\left[1 - \left(\dfrac{1-r}{1+r}\right)^{1-a}\right] \\ \text{[for } a < 1\text{]}, \\ \dfrac{2 - a}{4}\log\dfrac{1 + r}{1 - r} + \dfrac{a}{2}\dfrac{r}{(1 - r)^2} \\ \text{[for } 1 \leqq a \leqq 2\text{]}, \end{cases}$$

except that these bounds are reached for the functions w obtained upon replacing r by $ze^{\gamma i}$ and $-ze^{\gamma i}$ in the upper and lower bounds, respectively. When the region D is CONVEX, *then $a \leqq 1$, and we have*

$$\frac{1}{1 + 2ar + r^2} < \left|\frac{dw}{dz}\right| < \frac{1}{(1 + r)^{1+a}(1 + r)^{1-a}},$$

$$\left.\begin{matrix} \dfrac{1}{\sqrt{1 - a^2}}\arctan\dfrac{\sqrt{1 - a^2}.r}{1 + ar} \\ \text{[for } a < 1\text{]}, \\ \dfrac{r}{1 - r} \\ \text{[for } a = 1\text{]}, \end{matrix}\right\} < |w| < \begin{cases} \dfrac{1}{2}\log\dfrac{1 + r}{1 - r} \\ \text{[for } a = 0\text{]}, \\ \dfrac{1}{2a}\left[\left(\dfrac{1 + r}{1 - r}\right)^a - 1\right] \\ \text{[for } 0 < a \leqq 1\text{]}, \end{cases}$$

except for the functions w obtained as above.

As an application, let us suppose that the function

$$w = 1/z + c_1 z + \ldots + c_n z^n + \ldots$$

(without constant term) maps $|z| < 1$ on a simple region in the w-plane (this region containing, of course, $w = \infty$ in its interior). Inequalities of the form $|w| < k/r$ for $|z| = r$, $0 < r < 1$, k = constant, were obtained by Fricke[4] and the writer.[5] From the assumption, it follows that

$$W = 1/w = z + a_3 z^3 + \ldots + a_n z^n + \ldots$$

maps $|z| < 1$ on a simple region in the W-plane, and the application of our theorem with $a = 0$ gives

$$\frac{r}{1 + r^2} < |W| < \frac{r}{1 - r^2},$$

or

$$\frac{1}{r} - r < |w| < \frac{1}{r} + r$$

for $|z| = r$, $0 < r < 1$, except when

$$w = \frac{1}{z} + e^{2\alpha i}z$$

with α real, the upper and lower bounds of $|w|$ being then reached when $z = \pm e^{-\alpha i}r$ and $z = \pm ie^{-\alpha i}r$, respectively. This particular function maps $|z| < 1$ on the w-plane cut along the straight line from $w = -2e^{\alpha i}$ to $w = 2e^{\alpha i}$.

[1] Koebe, *Göttingen, Nachr. Ges. Wiss.*, 1909 (73).

[2] Gronwall, *Paris, C. R. Acad. Sci.*, **162,** 1916 (249).

[3] Gronwall, *Ibid.*, **162,** 1916 (316).

[4] Fricke and Klein, *Vorlesungen über die Theorie der automorphen Functionen*, II, Leipzig, 1912 (497).

[5] Gronwall, *Annals Math., Princeton*, Ser. 2, **16,** 1914 (74).

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*ON THE CONNECTION OF THE SPECIFIC HEATS WITH THE EQUATION OF STATE OF A GAS**

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Read before the Academy, April 26, 1920

One would suppose that this subject was already exhausted, but I am disposed to believe that such is not the case. In these PROCEEDINGS, **5**, July, 1919 (286–288) I published a paper "On the Possible Form of the Equation of State of Powder Gases." In a private letter from M. LeChatelier, in commenting on this paper, he says: "Votre objection au sujet de nos chaleurs spécifiques nous était venue à l'esprit, mais nous pensions avoir démontré, en partant des principes de la thermodynamique, que toutes les fois qu'un fluide obéit a une équation caracteristique de la forme

$$V = F(P/T)$$

les chaleurs spécifiques sont indépendantes de la pression."

I do not find where, if anywhere, M. LeChatelier has published this conclusion. Certainly it is not in the great paper by Mallard and LeChatelier, *Journal de Physique*, Ser. 2, **4** (59–84), "Recherches sur la Combustion des Mélanges Gazeux Explosifs," where the matter is not mentioned. In a paper published in the *Physical Review*, **29**, No. 3, September, 1909, "On the Definition of an Ideal Gas," written by me, but suggested by my then colleague Professor Rosanoff, and published under our joint names, I have investigated the properties of a gas having the equation of state mentioned by M. LeChatelier, but have said nothing about the specific heats. Inasmuch as the matter seems to be not quite

* Contribution from the Ballistic Institute, Clark University, No. 8.